

TOPOLOGICAL SET LABELING OF CERTAIN TADPOLE GRAPHS

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Abstract: The main objective of this paper is to compute the proper inequivalent τ' set labeling of certain tadpole graphs.

Keywords and Phrases: Topological number, Proper τ -set labeling, inequivalent topologies, Proper inequivalent τ' set labeling, Proper inequivalent τ' set labeling number.

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1. Introduction

B. D. Acharya created a link between graph theory and topology by introducing the topological set indexer concept. Later K. A. Germina extended it to topogenic graphs, topogenic index and graceful topogenic set indexers. Further N. K. Sudev and K. A. Germina discussed the concept of Topological IASL graphs. Inspired by their works we introduced topological set labeling concept.

2. Preliminaries

Definition 2.1. Let G be a graph. X be a non empty set whose cardinality is a topological number and τ denotes the set of all topologies of X . Then the set $f(v) = \{Y/Y \in \tau\}$ for which the set valuation $f : V \cup E \rightarrow 2^X$ is a t -set indexer of G is called proper τ -set labeling, otherwise it is called improper τ set labeling. The corresponding cardinality of $f(v)$ is called proper set labeling number and is denoted by η .

Definition 2.2. Let G be a graph. X be a non empty set whose cardinality is a topological number and τ denotes the set of all inequivalent topologies of X . Then the set $f(v) = \{Y/Y \in \tau\}$ for which the set valuation $f : V \cup E \rightarrow 2^X$ is a t -set indexer of G is called proper inequivalent τ' set labeling. The corresponding cardinality of $f(v)$ is called proper inequivalent τ' set labeling number and is denoted by η' .

Theorem 2.3. For a tadpole graph $T_{3,1}$ proper inequivalent τ' set labeling number is one.

Proof. Consider the base set N as $\{p, q, r\}$. Let τ' denotes the set of all inequivalent topology sets of N and τ'' be the sets in τ' having four elements. i.e. $\tau'' = \{\{\{\emptyset\}, \{r\}, \{q, r\}, \{p, q, r\}\}, \{\{\emptyset\}, \{r\}, \{p, q\}, \{p, q, r\}\}\}$. Let $f(q_{i1}) = \{r\}$, $f(q_{i2}) = \{\emptyset\}$, $f(q_{i3}) = \{p, q, r\}$, $f(q_{j1}) = \{q, r\}$ and $(q_{i1}, q_{i2}) = f(q_{i1}) \oplus f(q_{i2})$, $(q_{i2}, q_{i3}) = f(q_{i2}) \oplus f(q_{i3})$, $(q_{i3}, q_{i1}) = f(q_{i3}) \oplus f(q_{i1})$, $(q_{i1}, q_{j1}) = f(q_{i1}) \oplus f(q_{j1})$. Here $f : V \cup E \rightarrow 2^N$ satisfies the definition of t -set indexer. Labeling the other set $\{\{\emptyset\}, \{r\}, \{p, q\}, \{p, q, r\}\}$ to the vertex set of $T_{3,1}$ in any order will not satisfy the injective condition. Hence $\eta'(T_{3,1}) = 1$.

Theorem 2.4. $\eta'(T_{4,1})$ is 2.

Proof. Consider the base set as $N = \{p, q, r\}$. let τ' denotes the set of all inequivalent topologies on N and τ'' be the sets in τ' having 5 elements. i.e. $\tau'' = \{\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, q, r\}\}, \{\{\emptyset\}, \{r\}, \{q, r\}, \{p, r\}, \{p, q, r\}\}\}$.

Case 1: Consider the topology set $\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, q, r\}\}$. Let $\{q_{i1}, q_{i2}, q_{i3}, q_{i4}, q_{j1}\}$ be the vertex set and $\{(q_{i1}, q_{i2}), (q_{i2}, q_{i3}), (q_{i3}, q_{i4}), (q_{i4}, q_{j1}), (q_{i1}, q_{j1})\}$ be the edge set of $T_{4,1}$. Label the above set to the vertex set in any order, our definition of t -set indexer will be satisfied.

Case 2: By assigning distinct pair of elements of the set $\{\{\emptyset\}, \{r\}, \{q, r\}, \{p, r\}, \{p, q, r\}\}$ to the vertices and labeling edges by finding symmetric difference we could find that $\eta'(T_{4,1}) = 2$.

Theorem 2.5. Proper inequivalent τ' set labeling number of tadpole graph $T_{5,1}$ is 4.

Proof. Consider the graph $T_{5,1}$. Consider the base set $N = \{p, q, r, s\}$. Let τ' denotes the set of all inequivalent topologies of base set and τ'' be the sets in τ' having six elements. i.e. $\tau'' = \{\{\{\emptyset\}, \{p\}, \{p, q, r\}, \{s\}, \{p, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q, r\}, \{p, q, r\}, \{p, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p, q\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}, \{p\}\}, \{\{\emptyset\}, \{p\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, q, r\}, \{p, q, r, s\}\}\}$. Assign these sets as shown in following tables, required condition of t -set indexer definition will be satisfied.

Case 1:

Topology set	$f(V_{5,1})$	$f^\oplus E(T_{5,1})$
$\{\{\emptyset\}, \{p\}, \{p, q, r\}, \{s\},$ $\{p, s\}, \{p, q, r, s\}\}$	$f(q_{i1}) = \{p, q, r\}$ $f(q_{i2}) = \{p\}$ $f(q_{i3}) = \{\emptyset\}$ $f(q_{i4}) = \{p, s\}$ $f(q_{i5}) = \{p, q, r, s\}$ $f(q_{j1}) = \{s\}$	$f^\oplus(q_{i1}, q_{i2}) = \{q, r\}$ $f^\oplus(q_{i2}, q_{i3}) = \{p\}$ $f^\oplus(q_{i3}, q_{i4}) = \{p, s\}$ $f^\oplus(q_{i4}, q_{i5}) = \{p, q, r\}$ $f^\oplus(q_{i5}, q_{i1}) = \{s\}$ $f^\oplus(q_{i1}, q_{j1}) = \{p, q, r, s\}$

Case 2:

Topology set	$f(V_{5,1})$	$f^\oplus E(T_{5,1})$
$\{\{\emptyset\}, \{p\}, \{p, q\}, \{q\},$ $\{p, q, r\}, \{p, q, r, s\}\}$	$f(q_{i1}) = \{p, q\}$ $f(q_{i2}) = \{p\}$ $f(q_{i3}) = \{p, q, r, s\}$ $f(q_{i4}) = \{p, q, r\}$ $f(q_{i5}) = \{q\}$ $f(q_{j1}) = \{\emptyset\}$	$f^\oplus(q_{i1}, q_{i2}) = \{q\}$ $f^\oplus(q_{i2}, q_{i3}) = \{q, r, s\}$ $f^\oplus(q_{i3}, q_{i4}) = \{r\}$ $f^\oplus(q_{i4}, q_{i5}) = \{p, r\}$ $f^\oplus(q_{i5}, q_{i1}) = \{p\}$ $f^\oplus(q_{i1}, q_{j1}) = \{p, q\}$

Case 3:

Topology set	$f(V_{5,1})$	$f^\oplus E(T_{5,1})$
$\{\{\emptyset\}, \{p\}, \{p, q\}, \{p, r\},$ $\{p, q, r\}, \{p, q, r, s\}\}$	$f(q_{i1}) = \{p, r\}$ $f(q_{i2}) = \{p, q\}$ $f(q_{i3}) = \{p, q, r, s\}$ $f(q_{i4}) = \{p, q, r\}$ $f(q_{i5}) = \{\emptyset\}$ $f(q_{j1}) = \{p\}$	$f^\oplus(q_{i1}, q_{i2}) = \{q, r\}$ $f^\oplus(q_{i2}, q_{i3}) = \{r, s\}$ $f^\oplus(q_{i3}, q_{i4}) = \{s\}$ $f^\oplus(q_{i4}, q_{i5}) = \{p, q, r\}$ $f^\oplus(q_{i5}, q_{i1}) = \{p, r\}$ $f^\oplus(q_{i1}, q_{j1}) = \{r\}$

Case 4:

Topology set	$f(V_{5,1})$	$f^\oplus E(T_{5,1})$
$\{\{\emptyset\}, \{p, q\}, \{p\}, \{p, q, s\}, \{p, q, r\}, \{p, q, r, s\}\}$	$f(q_{i1}) = \{p\}$ $f(q_{i2}) = \{p, q\}$ $f(q_{i3}) = \{\emptyset\}$ $f(q_{i4}) = \{p, q, r\}$ $f(q_{i5}) = \{p, q, s\}$ $f(q_{j1}) = \{p, q, r, s\}$	$f^\oplus(q_{i1}, q_{i2}) = \{q\}$ $f^\oplus(q_{i2}, q_{i3}) = \{p, q\}$ $f^\oplus(q_{i3}, q_{i4}) = \{p, q, r\}$ $f^\oplus(q_{i4}, q_{i5}) = \{r, s\}$ $f^\oplus(q_{i5}, q_{i1}) = \{q, s\}$ $f^\oplus(q_{i1}, q_{j1}) = \{q, r, s\}$

Labeling the remaining sets to the vertices of $T_{5,1}$ in any order it does not satisfy the definition of t -set indexer. Therefore $\eta'(T_{5,1}) = 4$.

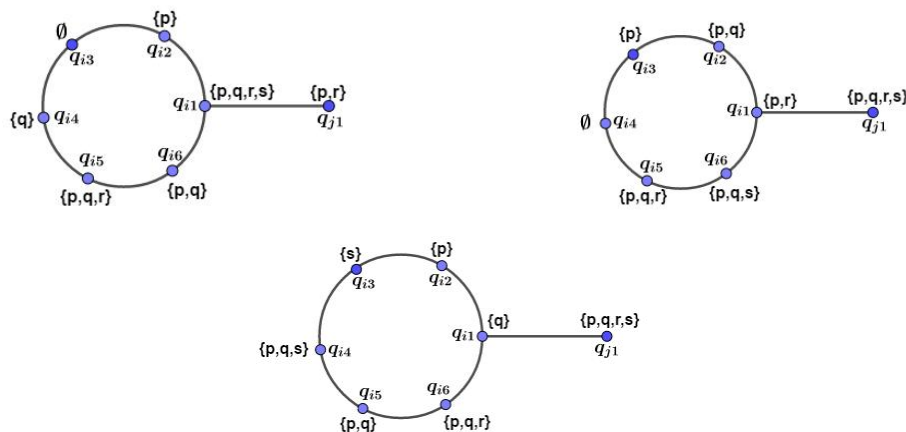
Theorem 2.6. For a tadpole graph with 6-cycles and 1-path, proper inequivalent τ' set labeling number is 3.

Proof. Consider the base set as $N = \{p, q, r, s\}$. Let τ' be the set of all inequivalent topologies on N and τ'' be the set in τ' having seven vertices. i.e. $\tau'' = \{\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}\}$

Case 1: Assign $\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, q, r, s\}\}$ as in the following figure which satisfies the definition of t -set indexer.

Case :2 Assign $\{\{\emptyset\}, \{p\}, \{p, q\}, \{p, r\}, \{p, q, s\}, \{p, q, r, s\}\}$ as in the following figure which satisfies the definition of t -set indexer.

Case 3: Assign the set $\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}$ as in following figure which satisfies the t -set indexer.



Theorem 2.7. $\eta'(T_{7,1}) = 2$.

Proof. Consider the base set $N = \{p, q, r, s\}$. Let τ' denotes the set of all inequivalent topologies of the set N and τ'' be the sets in τ' having eight elements. i.e. $\tau'' = \{\{\{\emptyset\}, \{p\}, \{p, q\}, \{r\}, \{p, r\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{r\}, \{p, q\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}\}$

Case 1: Let $\{q_{i1}, q_{i2}, \dots, q_{i7}, q_{j1}\}$ be the vertex set of $T_{7,1}$. Label $f(q_{i1}) = \{p, r\}$, $f(q_{i2}) = \{q\}$, $f(q_{i3}) = \{p\}$, $f(q_{i4}) = \{\emptyset\}$, $f(q_{i5}) = \{p, q, s\}$, $f(q_{i6}) = \{p, q, r, s\}$, $f(q_{i7}) = \{p, q\}$, $f(q_{j1}) = \{p, q, r\}$ which satisfies the definition of t -set indexer.

Case 2: Label the set $\{\{\emptyset\}, \{p\}, \{p, q\}, \{r\}, \{p, r\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}$ to the vertices of $T_{7,1}$ and the edges as $\{\{q, r, s\}, \{p, s\}, \{p, q\}, \{p, q, r\}, \{s\}, \{p, q, s\}, \{r, s\}, \{q, s\}\}$ suitably to form a t -set indexer. The set $\{\{\emptyset\}, \{p, q\}, \{r\}, \{p, q, r\}, \{s\}, \{p, q, s\}, \{r, s\}, \{p, q, r, s\}\}$ does not satisfy the condition of t -set indexer definition for $T_{7,1}$.

Theorem 2.8. Proper inequivalent τ' set labeling number of tadpole graph with 8-cycle and 1-path is 2.

Proof. Consider the base set as $N = \{p, q, r, s\}$. Let τ' denotes the set of all inequivalent topologies on N and τ'' be the sets in τ' having 9 elements. i.e. $\tau'' = \{\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{q, r\}, \{p, q, r\}, \{p, q, s\}, \{p, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, s\}, \{p, q, s\}, \{p, r, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{r\}, \{p, r\}, \{q, r\}, \{p, q, r\}, \{p, q, r, s\}\}\}$. Label these sets as in Table. Let Q denote the vertex set of $T_{8,1}$.

Table

Topology set	$Q(T_{8,1})$	$f : Q(T_{8,1}) \rightarrow 2^N$
$\{\emptyset\}, \{p\}, \{q\}, \{p, q\},$ $\{q, r\}, \{p, q, r\}, \{p, q, s\},$ $\{p, s\}, \{p, q, r, s\}$	$q_{i1}, q_{i2}, q_{i3}, q_{i4},$ $q_{i5}, q_{i6}, q_{i7},$ q_{i8}, q_{j1}	$\{q\}, \{p\}, \{q, r\}, \{\emptyset\},$ $\{p, s\}, \{p, q, s\}, \{p, q, r\},$ $\{p, q\}, \{p, q, r, s\}$
$\{\emptyset\}, \{p\}, \{p, q\},$ $\{p, q, r\}, \{p, s\},$ $\{p, r, s\}, \{p, q, r, s\}$ $\{p, r\}, \{p, s\}$ $\{p, q, s\}$	$q_{i1}, q_{i2}, q_{i3}, q_{i4},$ $q_{i5}, q_{i6}, q_{i7},$ q_{i8}, q_{j1}	$\{\emptyset\}, \{p, q, s\}, \{p\},$ $\{p, r, s\}, \{p, q\}, \{p, r\},$ $\{p, q, r\}, \{p, q, r, s\}$

Result. The set $\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{r\}, \{p, r\}, \{q, r\}, \{p, q, r\}, \{p, q, r, s\}\}$ does not satisfy the definition of t -set indexer for $T_{8,1}$.

Theorem 2.9. $\eta'(T_{9,1}) = 2$.

Proof. Consider the base set as $N = \{p, q, r, s\}$. Let τ' denotes the set of all inequivalent topologies on N and τ'' be the sets in τ' having 10 elements. i.e. $\tau'' = \{\{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{p, r\}, \{p, q, r\}, \{p, s\}, \{p, q, s\}, \{p, r, s\}, \{p, q, r, s\}\}, \{\{\emptyset\}, \{p\}, \{q\}, \{p, q\}, \{r\}, \{p, r\}, \{q, r\}, \{p, q, r\}, \{p, q, s\}, \{p, q, r, s\}\}\}$. Label the above sets as in the following figures respectively.



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